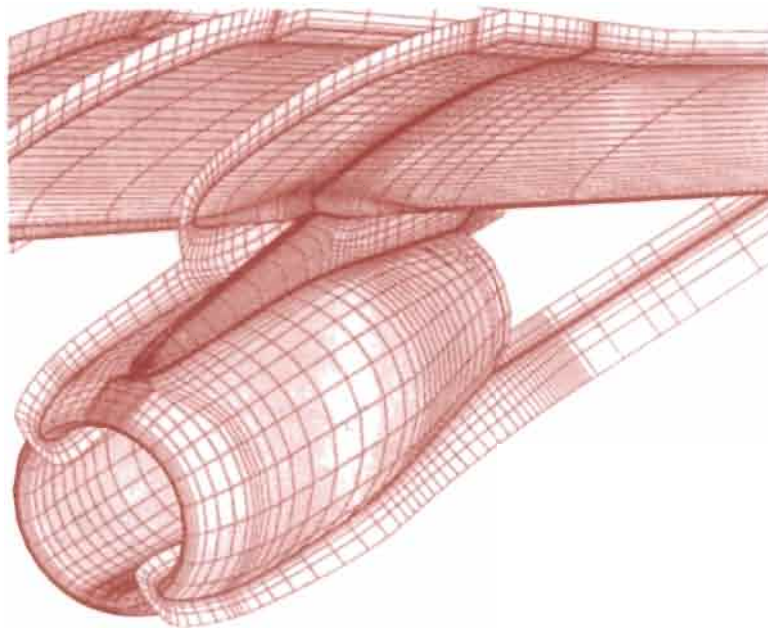


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J. Blazek

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Computational Fluid Dynamics: Principles and Applications

J. Blazek

*Alstom Power Ltd.,
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2001
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First edition 2001

Library of Congress Cataloging in Publication Data

A catalog record from the Library of Congress has been applied for.

British Library Cataloguing in Publication Data

Blazek, J. Computational fluid dynamics : principles and applications 1.Fluid dynamics - Computer simulation 2.Fluid dynamics - Mathematical models I.Title 532'.05 ISBN 0080430090

ISBN: 0 08 043009 0

♻ The paper used in this publication meets the requirements of ANSI/NISO Z39.48-1992 (Permanence of Paper).
Printed in The Netherlands.

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Acknowledgements

First of all I would like to thank my father for the initial motivation to start this project, as well as for his continuous help with the text and especially with the drawings. I thank my former colleagues from the Institute of Design Aerodynamics at the DLR in Braunschweig, Germany Norbert Kroll, Cord Rossow, Jose Longo, Rolf Radespiel and others for the opportunity to learn a lot about CFD and for the stimulating atmosphere. I also thank my colleague Andreas Haselbacher from ALSTOM Power in Daettwil, Switzerland (now at the University of Illinois at Urbana-Champaign) for reading and correcting significant parts of the manuscript, as well as for many fruitful discussions. I gratefully acknowledge the help of Olaf Brodersen from the DLR in Braunschweig and of Dimitri Mavriplis from ICASE, who provided several pictures of surface grids of transport aircraft configurations.

List of Symbols

\bar{A}_c	Jacobian of convective fluxes
\bar{A}_v	Jacobian of viscous fluxes
b	constant depth of control volume in two dimensions
c	speed of sound
c_p	specific heat coefficient at constant pressure
c_v	specific heat coefficient at constant volume
\vec{C}	vector of characteristic variables
C_m	molar concentration of species m ($= \rho Y_m / W_m$)
C_S	Smagorinsky constant
d	distance
D	diagonal part of implicit operator
\vec{D}	artificial dissipation
D_m	effective binary diffusivity of species m
e	internal energy per unit mass
E	total energy per unit mass
f	Fourier symbol of the time-stepping operator
\vec{f}_e	external force vector
\vec{F}	flux vector
$\overline{\overline{F}}$	flux tensor
g	amplification factor
h	enthalpy
H	total (stagnation) enthalpy
\bar{H}	Hessian matrix (matrix of second derivatives)
I	imaginary unit ($I = \sqrt{-1}$)
\bar{I}	identity matrix
$\bar{\bar{I}}$	unit tensor
\hat{I}_h^{2h}	interpolation operator

I_h^{2h}	restriction operator
I_{2h}^h	prolongation operator
\bar{J}	system matrix (implicit operator)
J^{-1}	inverse of determinant of coordinate transformation Jacobian
k	thermal conductivity coefficient
K	turbulent kinetic energy
K_f, K_b	forward and backward reaction rate constants
l_T	turbulent length scale
\mathbf{L}	strictly lower part of implicit operator
L_{ij}	components of Leonard stress tensor
M	Mach number
\bar{M}	mass matrix
\vec{n}	unit normal vector (outward pointing) of control volume face
n_x, n_y, n_z	components of the unit normal vector in x -, y -, z -direction
N	number of grid points, cells, or control volumes
N_A	number of adjacent control volumes
N_F	number of control volume faces
p	static pressure
\bar{P}	transformation matrix from conservative to primitive variables
Pr	Prandtl number
\dot{q}_h	heat flux due to radiation, chemical reactions, etc.
Q	source term
\vec{r}	position vector (Cartesian coordinates); residual (GMRES)
\vec{r}_{ij}	vector from point i to point j
R	specific gas constant
R_u	universal gas constant (= 8314.34 J/kg-mole K)
\vec{R}	residual, right-hand side
\vec{R}^*	smoothed residual
$\bar{\mathcal{R}}$	rotation matrix
Re	Reynolds number
\dot{s}_m	rate of change of species m due to chemical reactions
\vec{S}	face vector (= $\vec{n} \Delta S$)
S_{ij}	components of strain-rate tensor
S_x, S_y, S_z	Cartesian components of the face vector
dS	surface element
ΔS	length / area of a face of a control volume

t	time
t_T	turbulent time scale
Δt	time step
T	static temperature
\bar{T}	matrix of right eigenvectors
\bar{T}^{-1}	matrix of left eigenvectors
u, v, w	Cartesian velocity components
u_τ	skin friction velocity ($= \sqrt{\tau_w/\rho}$)
U	general (scalar) flow variable
\mathbf{U}	strictly upper part of implicit operator
\vec{U}	vector of general flow variables
\vec{v}	velocity vector with the components $u, v,$ and w
V	contravariant velocity
V_r	contravariant velocity relative to grid motion
V_t	contravariant velocity of control volume face
$\text{curl } \vec{v}$	$\text{curl of } \vec{v} \left(= \vec{\nabla} \times \vec{v} = \left[\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}, \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}, \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right] \right)$
$\text{div } \vec{v}$	$\text{divergence of } \vec{v} \left(= \vec{\nabla} \cdot \vec{v} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right)$
W_m	molecular weight of species m
\vec{W}	vector of conservative variables ($= [\rho, \rho u, \rho v, \rho w, \rho E]^T$)
\vec{W}_p	vector of primitive variables ($= [p, u, v, w, T]^T$)
x, y, z	Cartesian coordinate system
Δx	cell size in x-direction
y^+	non-dimensional wall coordinate ($= \rho y u_\tau / \mu_w$)
Y_m	mass fraction of species m
z	Fourier symbol of the spatial operator
α	angle of attack, inlet angle
α_m	coefficient of the Runge-Kutta scheme (in stage m)
β	parameter to control time accuracy of an implicit scheme
β_m	blending coefficient (in stage m of the Runge-Kutta scheme)
γ	ratio of specific heat coefficients at constant pressure and volume
Γ	circulation
$\bar{\Gamma}$	preconditioning matrix
δ_{ij}	Kronecker symbol
ε	rate of turbulent energy dissipation

ϵ	smoothing coefficient (implicit residual smoothing); parameter
κ	thermal diffusivity coefficient
λ	second viscosity coefficient
Λ_c	eigenvalue of convective flux Jacobian
$\bar{\Lambda}_c$	diagonal matrix of eigenvalues of convective flux Jacobian
$\hat{\Lambda}_c$	spectral radius of convective flux Jacobian
$\hat{\Lambda}_v$	spectral radius of viscous flux Jacobian
μ	dynamic viscosity coefficient
ν	kinematic viscosity coefficient ($= \mu/\rho$)
ξ, η, ζ	curvilinear coordinate system
ρ	density
σ	Courant-Friedrichs-Lewy (CFL) number
σ^*	CFL number due to residual smoothing
τ	viscous stress
τ_w	wall shear stress
$\bar{\tau}$	viscous stress tensor (normal and shear stresses)
τ_{ij}	components of viscous stress tensor
τ_{ij}^F	components of Favre-averaged Reynolds stress tensor
τ_{ij}^R	components of Reynolds stress tensor
τ_{ij}^S	components of subgrid-scale stress tensor
τ_{ij}^{SF}	components of Favre-filtered subgrid-scale stress tensor
τ_{ij}^{SR}	components of subgrid-scale Reynolds stress tensor
ω	rate of dissipation per unit turbulent kinetic energy ($=\epsilon/K$)
Υ	pressure sensor
Ω	control volume
Ω_{ij}	components of rotation-rate tensor
$\partial\Omega$	boundary of a control volume
Ψ	limiter function
$\vec{\nabla}U$	gradient of scalar U ($= \left[\frac{\partial U}{\partial x}, \frac{\partial U}{\partial y}, \frac{\partial U}{\partial z} \right]$)
$\nabla^2 U$	Laplace of scalar U ($= \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2}$)
$\ \vec{U}\ _2$	2-norm of vector \vec{U} ($= \sqrt{\vec{U} \cdot \vec{U}}$)

Subscripts

C	convective part
c	related to convection
D	diffusive part
i, j, k	nodal point index
I, J, K	index of a control volume
L	laminar; left
m	index of control volume face; species
R	right
T	turbulent
v	viscous part
V	related to volume
w	wall
x, y, z	components in the x-, y-, z-direction
∞	at infinity (farfield)

Superscripts

I, J, K	direction in computational space
n	previous time level
$n + 1$	new time level
T	transpose
\sim	Favre averaged mean value; Favre-filtered value (LES)
$''$	fluctuating part of Favre decomposition; subgrid scale (LES)
$-$	Reynolds averaged mean value; filtered value (LES)
$'$	fluctuating part of Reynolds decomposition; subgrid scale (LES)

Abbreviations

AIAA	American Institute of Aeronautics and Astronautics
AGARD	Advisory Group for Aerospace Research and Development (NATO)
ARC	Aeronautical Research Council, UK
ASME	The American Society of Mechanical Engineers
CERCA	Centre de Recherche en Calcul Applique (Centre for Research on Computation and its Applications), Montreal, Canada
CERFACS	Centre Europeen de Recherche et de Formation Avancee en Calcul Scientifique (European Centre for Research and Advanced Training in Scientific Computation), France
DFVLR	(now DLR) Deutsche Forschungs- und Versuchsanstalt für Luft- und Raumfahrt (German Aerospace Research Establishment)
DLR	Deutsches Zentrum für Luft- und Raumfahrt (German Aerospace Center)
ERCOFTAC	European Research Community on Flow, Turbulence and Combustion
ESA	European Space Agency
FFA	Flygtekniska Försöksanstalten (The Aeronautical Research Institute of Sweden)
GAMM	Gesellschaft für Angewandte Mathematik und Mechanik (German Society of Applied Mathematics and Mechanics)
ICASE	Institute for Computer Applications in Science and Engineering, NASA Langley Research Center, Hampton, VA, USA
INRIA	Institut National de Recherche en Informatique et en Automatique (The French National Institute for Research in Computer Science and Control)
ISABE	International Society for Air Breathing Engines

MAE	Department of Mechanical and Aerospace Engineering, Princeton University, Princeton, USA
NACA	(now NASA) The National Advisory Committee for Aero- nautics, USA
NASA	National Aeronautics and Space Administration, USA
NLR	Nationaal Lucht en Ruimtevaartlaboratorium (National Aerospace Laboratory), The Netherlands
ONERA	Office National d'Etudes et de Recherches Aérospatiales (National Institute for Aerospace Studies and Research), France
SIAM	Society of Industrial and Applied Mathematics, USA
VKI	Von Karman Institute for Fluid Dynamics, Belgium
ZAMM	Zeitschrift für angewandte Mathematik und Mechanik (Journal of Applied Mathematics and Mechanics), Germany
ZFW	Zeitschrift für Flugwissenschaften und Weltraumforschung (Journal of Aeronautics and Space Research), Germany
1D	one dimension
1-D	one-dimensional
2D	two dimensions
2-D	two-dimensional
3D	three dimensions
3-D	three-dimensional