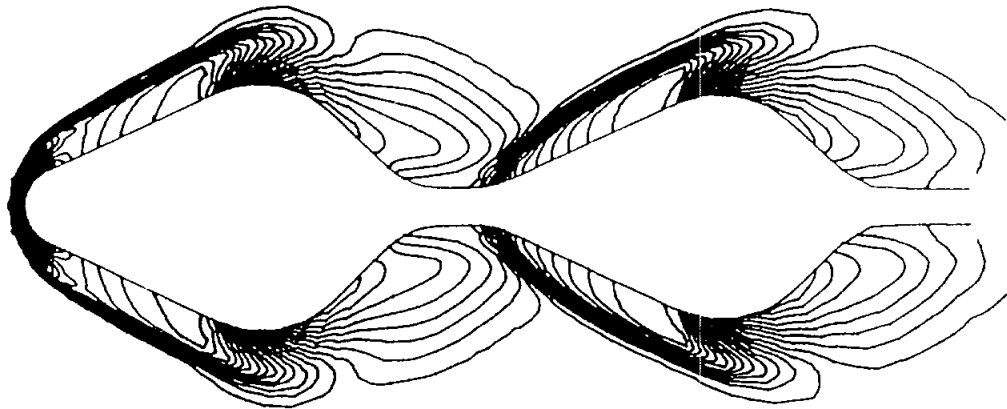


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**COMPUTATIONAL FLUID DYNAMICS**  
**VOLUME III**      **FOURTH EDITION**





Fourth Edition

COMPUTATIONAL FLUID DYNAMICS  
VOLUME III

KLAUS A. HOFFMANN

STEVE T. CHIANG

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# PREFACE

The fundamental concepts and development of computational schemes for the solution of parabolic, elliptic, and hyperbolic equations are established in Volume I and are extended to the Navier-Stokes equations in Volume II. The primary goal in the third volume is to review the fundamentals of turbulence and turbulent flows and to extend the governing equations and numerical schemes developed in Volume II to include turbulence.

This volume begins with the basic definitions and concepts in turbulence and turbulent flows. Subsequently, the modification of the governing equations and numerical schemes is introduced. There are three approaches by which turbulent flowfields may be computed. The first approach is based on the averaged Navier-Stokes equations either in the form of Reynolds-Averaged Navier-Stokes (RANS) equations or Favre-Averaged Navier-Stokes (FANS) equations. These formulations, along with several turbulence models and numerical considerations for the solution of equations, are presented in Chapter 21. The second and third approaches are the Large Eddy Simulations (LES) and the Direct Numerical Simulations (DNS), which are presented in Chapter 23. Since typical computations involved in turbulence and, in particular, in DNS, require higher-order schemes such as compact finite difference formulations, these formulations are introduced in Chapter 22.

Finally, a computer code based on the RANS equations and several turbulence models have been developed and included in the text *Student Guide for CFD—Volume III*.

Again, our sincere thanks and appreciation are extended to all individuals acknowledged in the preface of the first volume. Thank you all very much for your friendship and encouragement.

Klaus A. Hoffmann  
Steve T. Chiang

