



TUNCER CEBECI · JIAN P. SHAO  
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# Computational Fluid Dynamics for Engineers



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# Computational Fluid Dynamics for Engineers

From Panel to Navier–Stokes Methods  
with Computer Programs

With 152 Figures, 19 Tables, 84 Problems  
and a CD-ROM



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# Preface

History reminds us of ancient examples of fluid dynamics applications such as the Roman baths and aqueducts that fulfilled the requirements of the engineers who built them; of ships of various types with adequate hull designs, and of wind energy systems, built long before the subject of fluid mechanics was formalized by Reynolds, Newton, Euler, Navier, Stokes, Prandtl and others. The twentieth century has witnessed many more examples of applications of fluid dynamics for the use of humanity, all designed without the use of electronic computers. They include prime movers such as internal-combustion engines, gas and steam turbines, flight vehicles, and environmental systems for pollution control and ventilation.

*Computational Fluid Dynamics* (CFD) deals with the numerical analysis of these phenomena. Despite impressive progress in recent years, CFD remains an imperfect tool in the comparatively mature discipline of fluid dynamics, partly because electronic digital computers have been in widespread use for less than thirty years. The Navier–Stokes equations, which govern the motion of a Newtonian viscous fluid were formulated well over a century ago. The most straightforward method of attacking any fluid dynamics problem is to solve these equations for the appropriate boundary conditions. Analytical solutions are few and trivial and, even with today’s supercomputers, numerically exact solution of the complete equations for the three-dimensional, time-dependent motion of turbulent flow is prohibitively expensive except for basic research studies in simple configurations at low Reynolds numbers. Therefore, the “straightforward” approach is still impracticable for engineering purposes.

Considering the successes of the pre-computer age, one might ask whether it is necessary to gain a greater understanding of fluid dynamics and develop new computational techniques, with their associated effort and cost. Textbooks on fluid dynamics reveal two approaches to understanding fluid dynamics processes. The first is to devise useful correlations through a progression from demonstrative experiments to detailed experimental investigations that yield additional

understanding and subsequent improvement of the processes in question. The second is to solve simplified versions of fluid dynamics equations for conservation of mass, momentum and energy for comparatively simple boundary conditions. There is great advantage in combining both approaches when addressing complex fluid dynamics problems, but interaction between these two approaches has been limited until recently by the narrow range of useful solutions that could be obtained by analytic methods or simple numerical computations. It is evident, therefore, that any method for increasing the accuracy of computational methods by solving more complete forms of the conservation equations than has been possible up to now is to be welcomed. The numerical approaches of CFD have, in most cases, proven much more powerful than the closed-form analytical solutions of the past. As an example, the flow through the blade passage of a gas turbine is three-dimensional, and, even if we ignore the problem of modeling the behavior of turbulence, the corresponding equations can only be solved by numerical methods; even the inviscid flow in an axisymmetric engine intake cannot be calculated by purely analytic methods. Thus, without computational fluid dynamics, we cannot calculate detailed flow characteristics essential to improving understanding and supporting the design process.

It should be recognized that both experimental and computational fluid dynamics require resources. The cost of experiments in some cases can be prohibitive as, for example, with extensive flight tests of airplanes, full-scale tests of a gas turbine, or destructive testing of expensive components. In such cases, it may be possible to reduce the number of experimental tests by using CFD, since only a relatively small number of experiments are required to check the accuracy of the numerical results. Of course, the cost of obtaining accurate numerical solutions of differential equations may also be large for a complex flow, but still are usually much less than the cost of the additional experiments that would otherwise be required. In reality, the most cost-effective approach to solving a fluid dynamics problem is likely to be a combination of measurements and calculations. Both are subject to uncertainties, but the combination of these two approaches can result in a more cost-effective and more reliable design than by using only one approach or the other, and thus may be necessary to meet today's more stringent requirements for improved performance and reduced environmental impact, along with technical innovation and economy.

This book is an introduction to computational fluid dynamics with emphasis on the solution of conservation equations for incompressible and compressible flows with two independent variables. From the range of formulations in CFD, such as finite-difference, finite volume, finite element, spectral methods and direct numerical simulation, it concentrates on the first two, which are widely used to solve engineering problems. The restriction to two-dimensional flow and the omission of finite element, spectral methods and direct numerical simulation are imposed to facilitate understanding and to allow the essential material to be

presented in a book of modest size. The discussions, however, are general in this introductory book and apply to a variety of flows, including three-dimensional flows.

The format of the book assures that essential topics are covered in a logical sequence. The Introduction of Chapter 1 presents some examples to demonstrate the use of computational fluid dynamics for solving engineering problems of relevance. Chapter 2 presents the conservation equations; it is comparatively brief since detailed derivations are available elsewhere. The third chapter introduces important properties of turbulent flows, and exact and modeled forms of the turbulence equations with explanations to justify the assumptions of the models.

Chapters 4 and 5 provide an introduction to the numerical methods for solving the model equations for conservation equations which are useful for modeling the behavior of the more complete and complicated parabolic, hyperbolic and elliptic partial-differential equations considered in subsequent chapters. Chapter 4 discusses the numerical methods for the model parabolic and elliptic equations and Chapter 5 the model hyperbolic equations and include many computer programs.

The calculation of solutions for inviscid and boundary-layer equations is addressed in Chapters 6 and 7. Chapter 6 discusses finite-difference and panel methods for solving the Laplace equation and include computer programs for single and multi-element airfoils. Chapter 7 discusses the solution of laminar and turbulent boundary-layer equations for a prescribed external velocity distribution and specified transition location and includes a computer program based on Keller's finite-difference method.

The prediction of the onset of transition from laminar to turbulent flow has traditionally been achieved by correlations which are known to have limited ranges of applicability. The use of the  $e^n$ -method, based on the solutions of the stability equations, has been proposed as a more general approach. Chapter 8 describes the solution of the stability equations and provides a computer program for solving the Orr-Sommerfeld equation and computing transition with the  $e^n$ -method. It also presents applications of the stability/transition program, together with the computer programs of Chapters 6 and 7, to demonstrate how problems of direct relevance to engineering can be addressed by this approach.

Chapter 9 presents grid generation methods and is followed by Chapters 10 to 12 which describe methods for solving Euler (Chapter 10), incompressible Navier-Stokes (Chapter 11) and compressible Navier-Stokes equations. Again computer programs are included in each chapter and summarized in Appendix B.

A one semester course for advanced undergraduate and first-year graduate students would include a brief reading of Chapter 1 followed by Chapters 2, 4, 5 and 10 which include an extensive number of example problems and associated



computer programs arranged to provide the student a better understanding of the computational tools discussed. Parts of the material in Chapters 3, 6, 7 to 9 and 11 and 12 can be covered in a second semester course, with parts of the material in those chapters serving as useful information/reference.

A list of related and current books and solution manuals, including the one for the present book, published by Horizons and Springer-Verlag, is available on the Horizons Web site,

<http://hometown.aol.com/tuncerc/>

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Long Beach, April 2005

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