Fourth Edition

COMPUTATIONAL FLUID DYNAMICS
VOLUME I

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A Publication of Engineering Education System™, Wichita, Kansas, 67208-1078, USA

www.EESbooks.com

ISBN 0-9623731-0-9
First Print: August 2000

This book is typeset by Jeanie Duvall dba SciTech Computer Typesetting of Austin, Texas.

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PREFACE

This three-volume text is designed for use in introductory, intermediate, and advanced courses in computational fluid dynamics (CFD) and computational fluid turbulence (CFT). The fundamentals of computational schemes are established in the first volume, presented in nine chapters. The first seven chapters include basic concepts and introductory topics, whereas Chapters 8 and 9 cover advanced topics. In the second volume, the fundamental concepts are extended for the solution of the Euler, Parabolized Navier-Stokes, and Navier-Stokes equations. Finally, unstructured grid generation schemes, finite volume techniques, and finite element method are explored in the second volume. In the third volume, turbulent flows and several computational procedures for the solution of turbulent flows are addressed.

The first two volumes are designed such that they can be easily adapted to two sequential courses in CFD. Students with an interest in fluid mechanics and heat transfer should have sufficient background to undertake these courses. In addition, fundamental knowledge of programming and graphics is essential for the applications of methods presented throughout the text. Typically, the first course is offered at the undergraduate level, whereas the second course can be offered at the graduate level. The third volume of the text is designed for a course with the major emphasis on turbulent flows.

The general approach and presentation of the material is intended to be brief, with emphasis on applications. A fundamental background is established in the first seven chapters, where various model equations are presented, and the procedures used for the numerical solutions are illustrated. For purposes of analysis, the numerical solutions of the sample problems are presented in tables. In many instances, the behavior of a solution can be easily analyzed by considering graphical presentations of the results; therefore, they are included in the text as well. Before attempting to solve the problems proposed at the end of each chapter, the student should try to generate numerical solutions of the sample problems, using codes developed individually or available codes modified for the particular application. The results should be verified by comparing them with the solutions presented in the text. If an analytical solution for the proposed problem is available, the numerical solution should be compared to the analytical solution.

The emphasis in the first volume is on finite difference methods. Chapter 1 classifies the various partial differential equations, and presents some fundamental concepts and definitions. Chapter 2 describes how to achieve approximate representation of partial derivatives with finite difference equations. Chapter 3 discusses procedures for solving parabolic equations. Stability analysis is presented in Chapter 4. The order for Chapters 3 and 4 can be reversed. In fact, the results of stability analysis are required for the solution of parabolic equations in Chapter 3. The reason that the solution procedure of parabolic equations is developed first in Chapter 3 is to spread the computer code developments, since they require a substantial amount of time compared to other assignments. This will prevent the concentration of code development in the latter part of the course. Procedures for solving elliptic and hyperbolic partial differential equations are presented in Chapters 5 and 6, respectively. Chapter 7 presents a scalar model equation equivalent of the Navier-Stokes equations. In this chapter numerical algorithms are investigated to solve a scalar model equation which includes unsteady, convective, and diffusive terms.

The solution schemes established in the first seven chapters are extended to the solution of a system of partial differential equations in Chapter 8. In particular, the Navier-Stokes equations for incompressible flows in primitive variables, as well as vorticity-stream function formulations, are reviewed. Subsequently, the numerical schemes and specification of appropriate boundary conditions are introduced. Finally, Chapter 9 is designed to introduce the structured grid generation techniques. Various schemes, along with applications, are illustrated in this chapter.

While every attempt has been made to produce an error-free text, it is inevitable that some errors still exist. The authors would greatly appreciate the reader's input on any corrections, so that they may be incorporated into future printings. Furthermore, we would appreciate any comments and/or suggestions from the readers on the improvement of the text. Please forward your comments by mail to:

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In addition to this three-volume text, *Computational Fluid Dynamics,* a three-volume text, *Student Guide to CFD,* has been developed. The text, *Student Guide to CFD,* includes computer codes, description of input/output, and additional example problems. However, it is important to emphasize that computer code development is an important aspect of CFD, and that, in fact, one learns a great deal about the numerical schemes and their behaviour as one develops, debugs, and validates his or her own computer code. Therefore, it is important to state here that the computer codes provided in the text *Student Guide to CFD* should not be used as an avenue to replace that aspect of CFD and that code development must be an important objective of the learning process. However, these codes can be used as a basis upon which one may develop other codes, or the codes can be modified for other applications.

The authors greatly appreciates the support and help of many friends and colleagues — in particular, Dr. John Bertin and Dr. James Forsythe of the U.S. Air Force Academy, Dr. Walter Rutledge of Sandia National Laboratories; Dr. Dennis Wilson and Dr. Douglas Cline of The University of Texas at Austin; Dr. Shamoun Siddiqui of the Ministry of Defense, Pakistan; Mr. John Buratti of IBM; Mr. Shigeki Harada of Hewlett-Packard, Japan; Dr. Yildirim B. Suzen of University of Kentucky; Mr. Apichart Devahastin; Mr. Jean-Francois Dietiker; and Mr. Henri-Marie Damevin of Wichita State University. Furthermore, we are indebted to many of our students at The University of Texas, The Wichita State University, and those who have participated in various CFD correspondence and short courses offered by AIAA, EES, and ASME.

Finally, we greatly appreciate the efforts of Mrs. Karen Rutledge for editorial work, Mr. Tim Valdez for art work, and Ms. Jeanie Duvall for her skillful typing of the manuscript.

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